

Optics Laboratory Work

Digital Holography

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1. Objectives

The objectives of this optics laboratory session are:

- Learn how to digitally record the phase and amplitude distribution of a light beam (section 3 on "Digital holography").
- Learn how to generate arbitrary phase and amplitude distributions (section 4 on "Spatial light modulators").
- Apply these techniques to model and control light propagation through a seemingly random optical system (section 5 on "Transmission matrix of a multimode fiber").

It is essential that you prepare this lab session in advance by reading through the lab notes. The tasks denoted with a symbol "□" are short questions that require a written preparation before the lab (a handwritten draft is sufficient). In addition, there will be an oral examination during the first session about the methods described in these notes.

Make sure to take notes and save images during the lab session, so that you can include this material in your final lab report.

If you need more information about certain topics of this lab session, feel free to consult the references given at the end of this document. If you need information about MATLAB commands, use the *help* function.

2. Safety

The laser source used in this lab is a class 3B laser, emitting approximately 100mW of visible light at 532nm. A number of precautions have been taken for your safety, such as attenuation and expansion of the laser output. The actual laser power in the experimental setup should therefore be eye-safe (below 1mW). You should nonetheless observe the following rules:

- Never look into a direct or reflected laser beam.
- Do not wear watches, rings or any other object that can reflect the laser beam.
- Keep eyes level higher than laser level, i.e. avoid sitting on the chair while the laser is on.
- Optical fibers are made of glass. Be careful during fiber manipulation.
- Laser safety goggles are available. You may use them for extra safety.

Additionally, please note that the experimental setup is pre-built and mostly pre-aligned. Do not carry out alignments or any other optics manipulation yourself, unless specifically instructed in these lab notes or by the teaching assistant. Specifically, do not unscrew or change the orientation of any of the components unless instructed. Do not tamper with the laser source or the single-mode fiber coupling box.

3. Digital holography

3.1. Introduction

In optics, the term "holography" refers to methods for recording both the amplitude and the phase distribution of a light field. It can be thought of as an extension of photography. Photographs record the two-dimensional intensity distribution of a light field (for multiple wavelengths in color photographs); holograms record both the phase as well as the amplitude at each point.

Recording the phase is useful in a number of applications. For example, in biology, cells are usually transparent, but they can be made visible by a phase recording due to their refractive properties. In material science, the phase recordings can reveal stresses and deformations of just a few tens of nanometers. Finally, holograms allow a number of manipulations that are impossible to do with normal photographs, such as digitally propagating the light field in space or lensless imaging.

Like photographs, holograms have traditionally been recorded inside photosensitive materials. The hologram was then "reconstructed" by illuminating this recording, thereby re-creating the original light field exactly as it came from the object. In this lab we will study a more modern implementation, namely digital holography, where the holographic recording is made on a CMOS sensor and processed on a computer. Numerous methods of digital holography exist, but we focus here on the particular case of off-axis digital holography.

3.2. Off-axis digital holography

In off-axis digital holography, the light wave to be recorded (the "object" wave) is superimposed with a plane wave (the "reference" wave) coming in at a certain angle relative to the optical axis. The resulting interference pattern is recorded on a camera sensor. This geometry is illustrated in Figure 1.

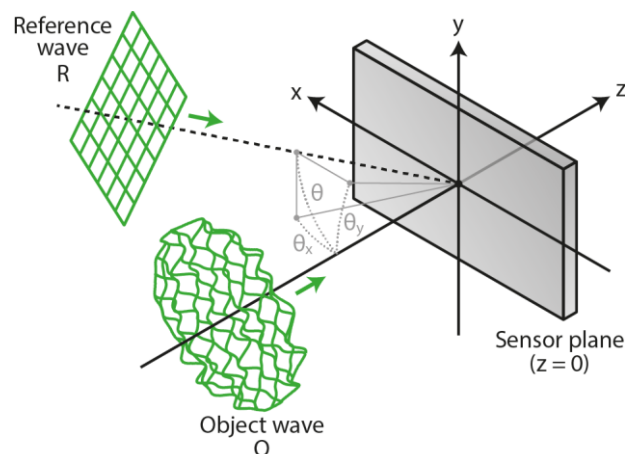


Figure 1: Geometry of off-axis digital holography

The typical interference pattern of an off-axis hologram is shown in Figure 2(a). In this example, the object wave is a random speckle pattern coming from an optical fiber. After taking the two-dimensional Fourier transform of this hologram, as in Figure 2(b), one can distinguish multiple separate regions. One of these regions contains the desired amplitude and phase information about the object wave. After isolating this term (Figure 2(c)) and correcting its location in the

Fourier domain (Figure 2(d)), the phase and amplitude distribution can be retrieved by inverse Fourier transform (Figure 2(e)).

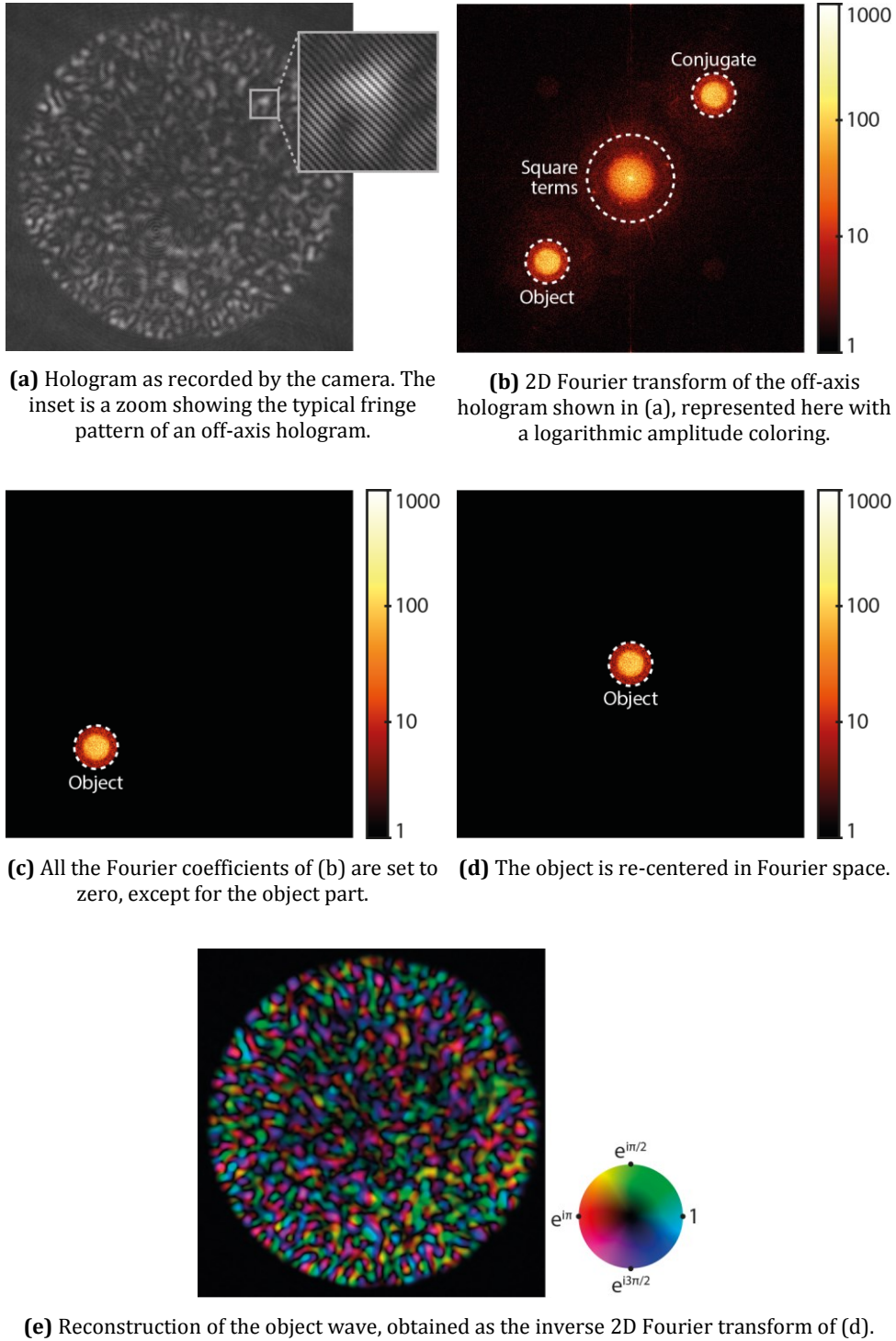


Figure 2: Digital reconstruction of an off-axis hologram

🏠 **Task 1: Answer the following questions.**

- a) The camera sensor detects the sum of the object wave O and the reference wave R . Write the equation for the intensity I obtained on the camera sensor. Show the appearance of four terms. Briefly describe the meaning of each term. For this calculation, note that R and O are complex quantities.
- b) Set the reference wave equal to an off-axis plane wave: $R(x, y) = e^{i k_0 (\sin \theta_x x + \sin \theta_y y)}$. Here, the magnitude of the wave vector is $k_0 = 2\pi/\lambda_0$ and the angles of this wave with the x- and y- axis respectively are θ_x and θ_y . Note that we neglected the time dependence by setting $t = 0$, and we also assumed the camera plane is at $z = 0$. Assume a general form for the object wave: $O = O(x, y)$. Now plug these waves in the expression you found in a) and take the two-dimensional Fourier transform. Determine the location of each term in frequency space by using the Fourier shift property:

$$\mathcal{F}\{e^{i(ax+by)} f(x; y)\}(k_x, k_y) = \hat{f}(k_x - a; k_y - b)$$

- c) Assume further, for simplicity, that the reference wave has an angle only in x (i.e. $\theta_y = 0$), and the object wave is a plane wave coming in at perpendicular angle on the camera sensor, i.e. $O(x, y) = 1$. The sum of O and R creates a periodic intensity pattern on the camera. What is the spacing d between the oscillations (period) of this pattern?
- d) A camera with a pixel size of L_{pixel} can correctly resolve a pattern only if its spatial period d is larger than two pixels: $d > 2 L_{\text{pixel}}$. This is the Nyquist-Shannon sampling criterion. It is schematically illustrated in Figure 3. Under this condition, what is the maximum allowable angle θ_x of the off-axis reference beam? Calculate this angle in degrees for a wavelength of 532nm and a camera with 8μm pixels.

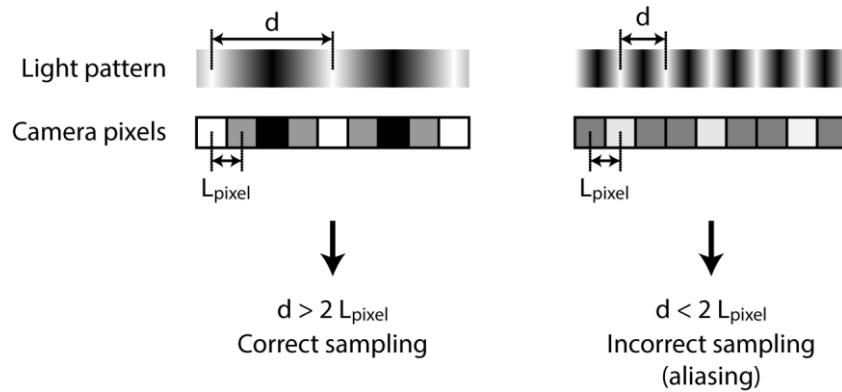


Figure 3: Sampling of a periodic fringe pattern with a pixelated device

3.3. Experiments

The experimental setup for the digital holography experiments is shown in Figure 4. Laser light is brought to the setup by the single mode fibers SMF1 and SMF2.

The illumination beam originates from SMF1, is polarized by filter LP1, collimated by lens L4, and is then directed via flip mirror FM1 and adjustable mirror KM3 to the Sample (i.e. the "object"). This object wave is then relayed via lens L3, mirrors KM4 and FM2, and lens L2 onto the surface of the Camera.

The off-axis reference beam originates from the second fiber SMF2, is polarized by LP2, collimated by L5, relayed by mirrors KM1 and KM2, and superposed with the object beam through a 50:50 beam splitter BS. Mirrors KM1 and KM2 allow adjusting the off-axis angle of the reference beam.

Note that in off-axis holography, it is not strictly necessary to combine the reference beam and the object beam via a beam splitter as is done here. The off-axis angle usually needs to be small, however, and without beam splitter we would need a very large distance in front of the camera before the object beam and the reference beam can be combined without their respective optical paths obstructing each other.

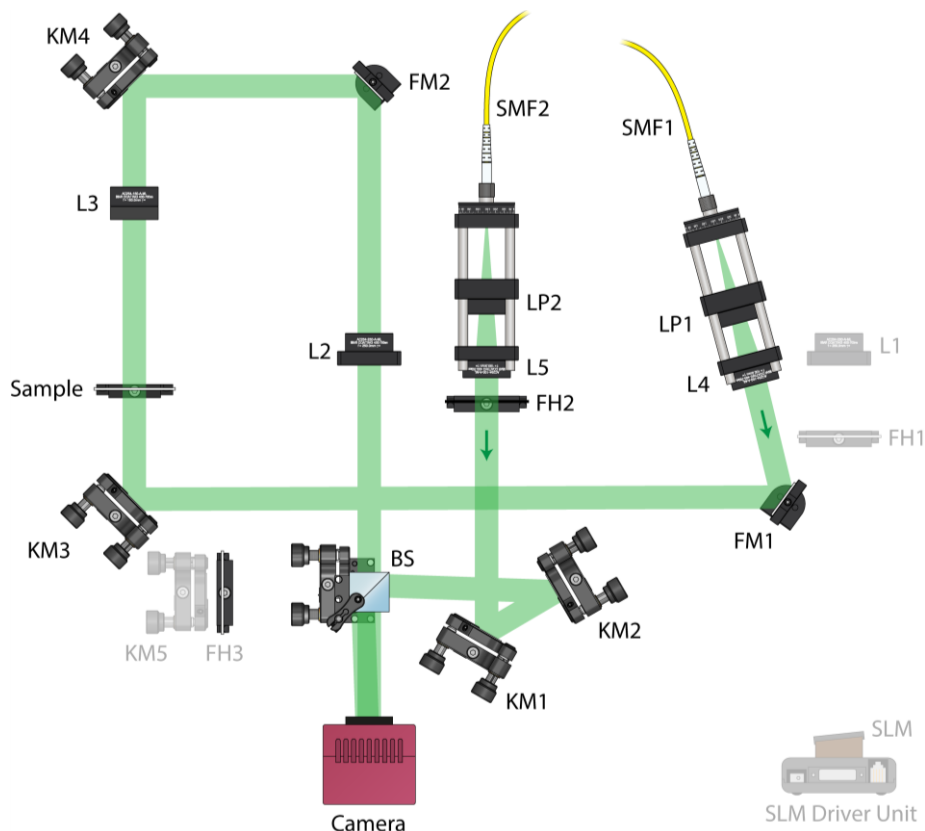


Figure 4: Setup for the digital holography experiments

Task 2: Record an off-axis digital hologram.

On the computer, start MATLAB by using the shortcut on the desktop.

In the setup, start with the flip mirror FM1 flipped down and flip mirror FM2 flipped up.

Do not touch the mirror surfaces. Flip the mirrors by holding the mount (black part) only.

For safety, make sure filter holder FH3 (refer to Figure 4) contains a piece of paper or cardboard blocking the light beam out of the left port of the beam splitter.

- a) Only one of the two beams is currently reaching the camera. Which one is it: the object beam or the reference beam? Observe the image recorded by the camera typing the command "camera" into the MATLAB command window. The camera tool has the following features:

- Save images for your report by pressing the key "s" from the camera window.
- Use "i" to import the current image in the MATLAB workspace.
- Use "+" and "-" to tune the exposure time.
- Press "f" to view the Fourier transform of the camera image in real time.
- Press "ESC" to close the tool. You cannot type MATLAB commands while the camera tool is running.
- Overexposed pixels are shown in red.

You are expected to include in your report an image with a brief description for each step of this experiment.

- b) Flip mirror FM1 up. Block the beam coming out of SMF2 by placing a piece of black cardboard in filter holder FH2. Look at the camera image. Which beam are you observing now on the camera? Briefly describe what you see.
- c) Remove the black cardboard from FH2. Both object and reference beam are now superposed on the camera. Observe the camera image and describe briefly.
- d) Press "f" in the camera window to observe the Fourier domain. Describe briefly. Press "f" again to switch back to the spatial domain.
- e) Now, change the angle of the reference beam by turning the knobs on adjustable mirror KM2. Turn the top knob over approximately 360° , in the counterclockwise direction as viewed from the back of the mount (refer to Figure 5). Do the same for the bottom knob. Observe the camera and its Fourier domain while you do this. Briefly describe what is happening (both in the camera image and in its Fourier transform).

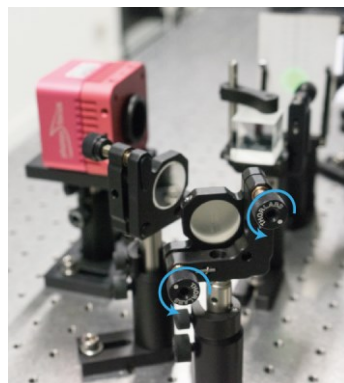


Figure 5: Adjustment of mirror KM2

- f) After the off-axis angle is adjusted, press "i" in the camera tool to import the current camera image (i.e. the hologram) into the MATLAB workspace, and proceed to Task 3.

Task 3: Digitally reconstruct the off-axis hologram.

- a) Observe the sample in the experimental setup. What does it look like by eye?
- b) Write a MATLAB script to reconstruct your hologram and retrieve the amplitude and phase profile of the sample. Use the procedure outlined in section 3.2 and Figure 2. Include each step in your report. The following functions are at your disposal if necessary:

- `img = fft2s(img)`
Takes the 2D Fourier transform of image *img*, and shifts the zero frequency towards the center of the resulting image.
- `img = ifft2s(img)`
Inverse Fourier transform of an image that was transformed with `fft2s`.
- `m = mask_circular(dims, x0, y0, r)`
Creates a circular mask with radius *r* and center (*x0*; *y0*) inside an image of size *dims* (*dims* is a vector with 2 elements, where the first number is the width and the second number is the height).
- `img = move_xy(img, dx, dy)`
Moves an image over the pixel distance *dx* and *dy*. Can also do sub-pixel shifts.
- `img = modulate_xy(img, dpx, dpy)`
Applies a linear phase modulation with spatial periods *dpx* and *dpy*. It can be useful to compensate residual phase oscillations over your reconstructed image.
- `abs(img)`, `angle(img)` and `db(img)`
Calculates the amplitude (*abs*), phase (*angle*) or logarithmic amplitude (*db*) of the complex image *img*.
- `imagesc(img)`
Displays an image on the screen (*imagesc*). It cannot display complex images, so for these images you first need to use *abs*, *angle* or *db*.
- `imwrite(img, filename)`
Writes image *img* to the file with name *filename* (e.g. "reconstructed_phase.png").
- `img_rgb = img_amp_phase(img)`
Converts a complex image *img* into a color image, where the brightness of each pixel represents the amplitude and the color represents the phase. The resulting image can be displayed with *imagesc* or saved using *imwrite*.

- c) The sample you recorded holographically is a piece of glass where grooves of a certain depth *d* have been etched on the surface. This is illustrated in Figure 6. Because the light travels through a different thickness of glass when passing inside the grooves versus outside the grooves, a phase difference is observed in the hologram. The depth of the groove can be estimated from this phase difference using the following formula:

$$\Delta\varphi = \varphi_2 - \varphi_1 = \frac{2\pi}{\lambda} d(n_{\text{glass}} - n_{\text{air}})$$

Estimate the depth of one of the grooves in your reconstructed hologram by comparing the phase at a point inside one of the grooves with a point just outside this groove. Assume that $n_{\text{glass}} = 1.45$. For accuracy, make sure the background phase modulation of your phase image is as low as possible.

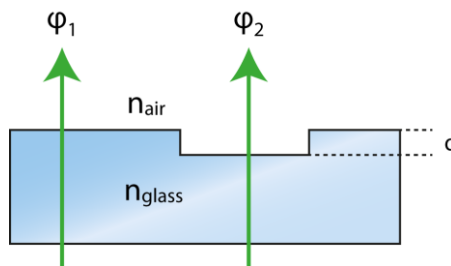


Figure 6: Illustration of the grooves in the glass sample

4. Spatial light modulators

4.1. Introduction

A spatial light modulator (SLM) is a device that can modulate an incoming light beam and impart a controllable two-dimensional spatial wave profile to this beam, as shown in Figure 7.

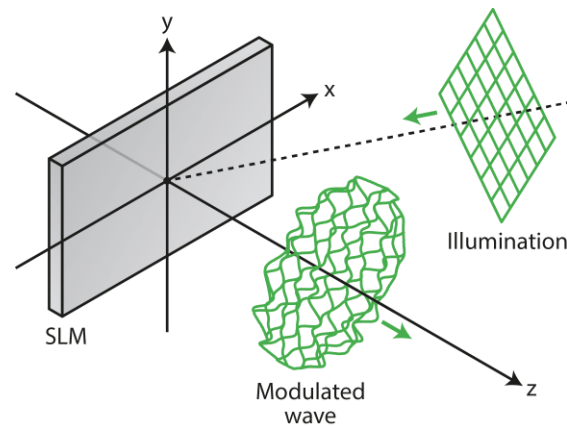


Figure 7: Illustration of a reflective SLM

An ideal SLM would modulate the phase and amplitude with high speed, resolution and accuracy. Practical SLMs are limited by current technology and usually fulfil only part of these criteria. In this lab, we use a phase-only liquid crystal modulator. This is a two-dimensional array of millions of liquid crystal cells (pixels). The orientation of the liquid crystals inside each cell can be independently controlled, thereby varying the phase delay of the light travelling through the cell, as shown in Figure 8. The result is a device that can create 2D distributions of phase.

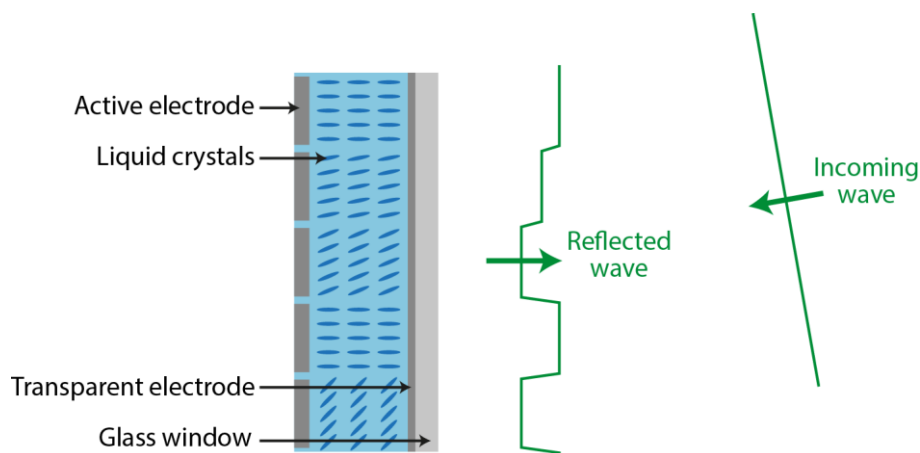


Figure 8: Schematic illustration of a liquid crystal modulator

4.2. Experiments

The setup for the SLM experiments is shown in Figure 9. The light beam coming out of fiber SMF1 is filtered by polarizer LP1, collimated by L4, and directed on the SLM. A viewing card can be placed in filter holder FH1 in order to observe the patterns reflected by the SLM.

The specifications of the SLM are shown in Table 1. The SLM is connected as a "second monitor" to the computer. To display phase distributions, it is necessary to open a window on this second monitor and display the phase as an 8-bit grayscale image (gray level 0 is phase 0°, gray level 128 is phase 180°, etc.). MATLAB functions are provided to do these steps for you.

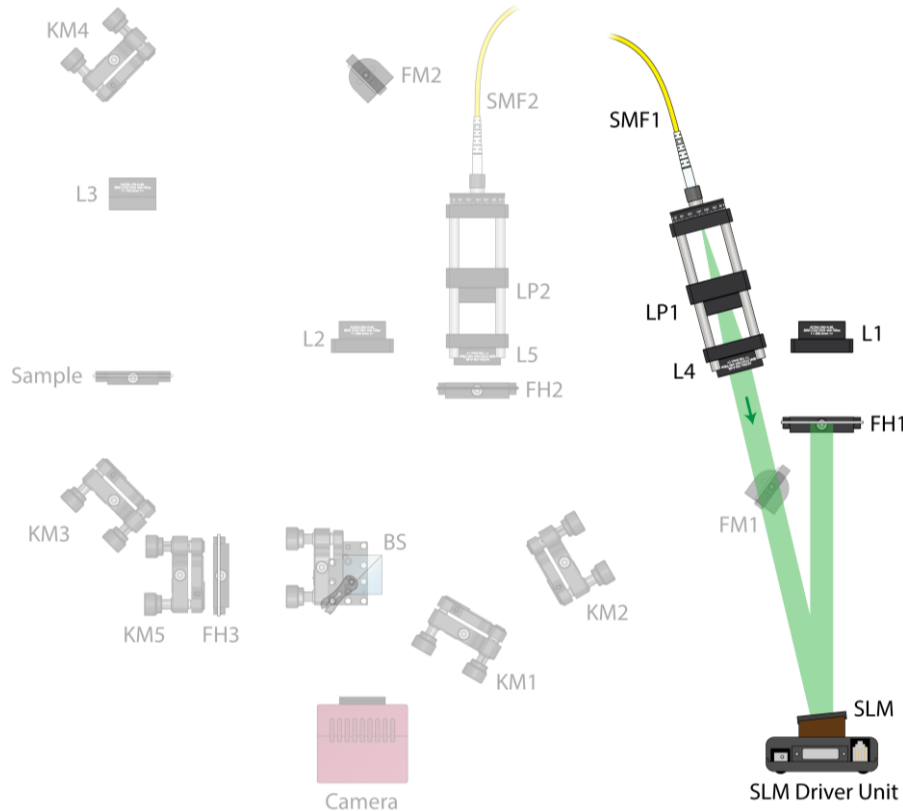


Figure 9: Setup configuration for the SLM experiments

Resolution	1920 x 1080 pixels (width x height)
Pixel pitch	8.0 μm
Fill factor	93%
Active area	15.36 mm x 8.64 mm
Bit depth	8 bit (256 gray levels)
Video frame rate	60 Hz

Table 1: SLM specifications

Task 4: Spatial light modulator.

Flip mirror FM1 down. Place a viewing card in filter holder FH1. The light reflected from the SLM onto the viewing card in FH1 is a homogeneous light field in the shape of the SLM's active area.

- a) In MATLAB, create an array of x and y coordinates representing the spatial coordinates of each pixel on the SLM.
- You can generate grids of coordinates using the MATLAB function *meshgrid*.
 - In MATLAB images, the first dimension is the y -axis and the second dimension is the x -axis. Therefore, your MATLAB arrays must have a size of 1080×1920 elements.
 - Place the origin (zero) in the middle of the grid (i.e. at element (541, 961)).
 - Express the grid coordinates in physical units (i.e. meters, not pixels).
- b) Compute a phase mask that simulates a lens of focal length $f = 200$ mm using the following lens phase formula:

$$F = e^{i \frac{2\pi}{\lambda} \frac{x^2 + y^2}{2f}}$$

Use the coordinates calculated in a). Display your phase mask on the SLM with the function *slm_display*. Would an actual lens with $f = 200$ mm create exactly the same pattern?

- c) Estimate the angular separation between the orders of the pattern you obtained in b). Do this by measuring the distance between the orders on the viewing card in FH1, and calculate the angle by trigonometry knowing that the distance to the SLM is approximately 200 mm.

Suppose the diffraction pattern you see was created by a periodic feature, what would be the size (period) of this feature (use the equation from Question 1c)? Using this value, look at the specifications of the SLM in Table 1 and determine the likely origin of these diffraction patterns (assume a certain margin of error on your angle measurement). Now explain briefly why the SLM creates such a pattern.

Would diffraction still occur if the SLM was turned off? Disconnect the SLM together with the teaching assistant and verify. Turn off the light in the room if no pattern is evident.

- d) Try various focal lengths f between 100 mm and 200 mm. Take out the viewing card from the filter holder FH1 and look at different positions in front of the SLM. **Do not touch the SLM panel.** Comment briefly. Place the viewing card back in FH1 when done.
- e) Compute a linear phase mask with the following formula:

$$F = e^{i 2\pi (p_x x + p_y y)}$$

The parameters p_x and p_y are the spatial frequencies (i.e. oscillations per meter) in the x - and y -direction respectively. Try a few values for the spatial frequencies p_x and p_y between $1\,000\text{ m}^{-1}$ and $100\,000\text{ m}^{-1}$. Display on the SLM and comment briefly.

- f) Multiply a lens phase mask (use $f = 200$ mm) with a linear phase mask (choose p_x and p_y freely), and observe what happens with on the viewing card in filter holder FH1.
- g) When you display the patterns on the SLM, does all of the light get modulated? Turn off the lights around the experiment to see this more clearly. Give one possible reason based on the device structure shown in Figure 8.
- h) Type the command "slm_demo" in MATLAB. Move the viewing card at various locations in front of the SLM. Observe that the SLM can create images at different locations in three-dimensional space. Also note that a pure phase SLM can be used to generate amplitude patterns under certain conditions.

5. Transmission matrix of a multimode fiber

5.1. Optical fibers

Optical fibers are waveguides made from two layers of glass (core and cladding), and they guide light by total internal reflection (see Figure 10).

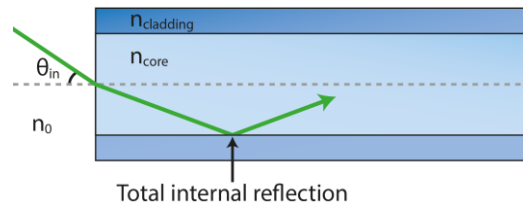


Figure 10: Total internal reflection in optical fibers

One distinguishes single-mode fibers and multi-mode fibers. Single-mode fibers accept only a single spatial distribution of light (i.e. a single mode; usually in the shape of a Bessel beam). Multi-mode fibers can guide a variety of light patterns or modes (i.e. multiple modes).

While multi-mode fibers accept two-dimensional light patterns, they tend to scramble the spatial information due to a phenomenon known as modal scrambling. This means that, unlike a pair of lenses, multi-mode fibers cannot directly relay images. This is illustrated in Figure 11. Modal scrambling is due to the particular structure of light propagation in fibers. Different modes travel at different phase velocities inside the fiber, causing them to lose their initial phase relationship with each other as they propagate.

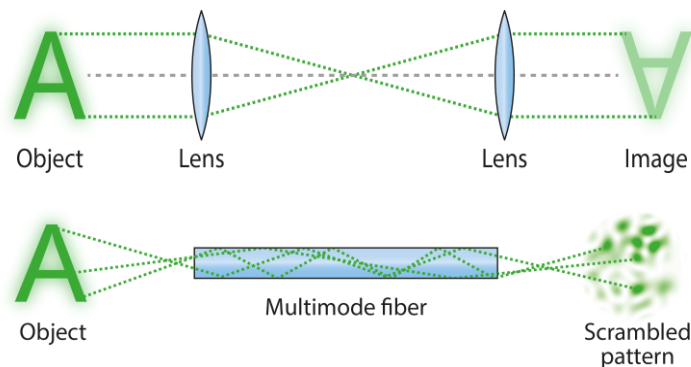


Figure 11: Fibers guide light but scramble the spatial structure of images.

For this reason, many practical imaging devices based on fibers (e.g. fiber endoscopes) contain *bundles* of thousands of separate fibers. Each fiber captures "one pixel" of the target location and relays it to the observer, independently from the other fibers. Using fiber bundles has a negative impact on resolution and probe size, however.

In the following experiments, we will use the spatial light modulator and digital holography to transmit images through one multimode fiber. Indeed, while modal scrambling looks random, it is nonetheless a deterministic process and it can be compensated. The method we will use for this is the transmission matrix.

5.2. Transmission matrix

The transmission matrix is a way to relate the input and the output of an optical system by a simple linear matrix operation $Y = T \cdot X$, where X is the input, Y is the output, and T is the transmission matrix. This is illustrated in Figure 12.

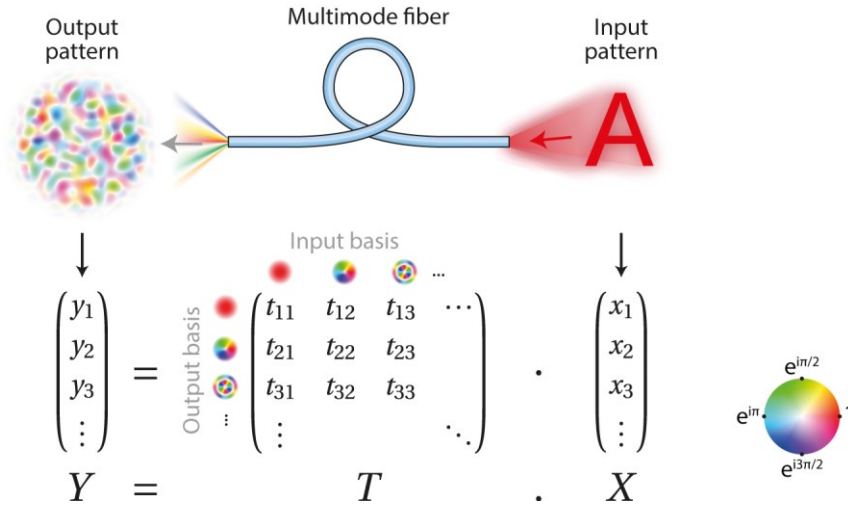


Figure 12: The transmission matrix model for a multimode fiber.

Such a model is possible because, assuming that all materials in the optical system are linear, Maxwell's equations are also linear. Therefore, the input-output response of the system obeys the superposition principle, and the rules of linear algebra apply. If the optical system has a finite number of modes, the transmission matrix is also of finite size. This is the case for all optical systems with inputs and outputs of finite spatial extent, including fibers.

Transmission matrices can be measured by applying a set of known inputs X_{test} to the system (for example using an SLM), and recording the output responses Y_{test} for each of those inputs (for example using digital holography). The matrix T can then be inferred by solving $Y_{test} = T \cdot X_{test}$ for T with a regularized matrix inversion algorithm.

There is not a unique transmission matrix for any given optical system. The particular form of the matrix depends on how the optical fields are encoded into vectors of complex numbers. In terms of algebra, this is called the "basis" of the matrix. For example, in Figure 12, a basis of fiber modes is used. This means that each coefficient $x_1, x_2, x_3, \dots$ and $y_1, y_2, y_3, \dots$ represents the weight of a particular fiber mode in the input or output field respectively. Another example is a pixel basis, where the complex field values (of the input or the output) over a two-dimensional grid of pixels are simply stored as subsequent elements in a one-dimensional column vector.

5.3. Calculation

The transmission matrix is conceptually simple, but its implementation in practice can be technically challenging due to the volume of data that needs to be processed. For this reason, this part of the lab is split in two tasks. The first task is a simple numerical exercise showing the various steps in a transmission matrix experiment. The second task is the experimental measurement, which is largely automated for your convenience.

🏠 Task 5: Numerical simulation of a matrix measurement.

Assume a simple optical system consisting of a fiber with 2 modes. At the input, there is an SLM with only 2 pixels. At the output, a camera records 2 pixels by holography (phase and amplitude). To find the matrix of the fiber, we display two linearly independent test inputs on the SLM. For each input, we record the corresponding output on the camera. The resulting data are shown in Table 2.

Input field displayed on the SLM	Output field recorded on the camera
1	$1+7i$
i	$-3-i$
1	$1+3i$
$-i$	$-3+3i$

Table 2: Data for the numerical transmission matrix simulation

- In MATLAB, create the matrices X_{test} and Y_{test} containing the test patterns along successive columns.
- Calculate the transmission matrix T .
- Calculate which input pattern needs to be displayed on the SLM in order to obtain the following output:

3	-3
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- Considering that the SLM is a phase-only device, is it possible to display the input pattern you calculated in c) ? Discard the amplitude information of this input pattern (i.e. set the amplitude of each pixel to 1), and calculate which output you would actually obtain with a phase-only input.

The experimental setup for the transmission matrix is shown in Figure 13. Light originates from SMF1, is collimated by L4 and then directed to the SLM. The pattern created by the SLM is imaged through the relay system composed of lenses L1 ($f = 250$ mm) and OBJ1 ($f = 3$ mm), with a demagnification factor of approximately 83. This pattern then travels through the multimode fiber MMF, whose tip is fixed in fiber clamp FC1. A quarter-wave plate QWP1 changes the polarization to circular, as this polarization is better preserved in step-index fibers. The aperture AP is used to block unwanted extra diffraction orders from the SLM. In the middle of the fiber, an adjuster PA allows to slightly change the bending state of the fiber (see later).

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Task 6: Experimental matrix measurement

- a) ☐ The multimode fiber MMF that is installed in the setup is a step index fiber with a core diameter of 50 μm and a numerical aperture of 0.22. Using the definition of the V-number and the formula for the number of modes M of a step index fiber given below, estimate the minimum size of the transmission matrix of this fiber (number of rows and columns). Assuming each element of the matrix takes 16 bytes of computer memory, how much memory will the matrix require? By what factor does the memory size increase if you double the size of the core or the numerical aperture? In the formulas below, λ is the laser wavelength, r is the core radius, and NA is the numerical aperture.

$$V = \frac{2\pi}{\lambda} r NA \qquad M \approx \frac{V^2}{4}$$

- b) Flip both mirrors FM1 and FM2 down.
Make sure that filter holders FH1 and FH2 are empty (not blocking the beam).

Start the script "script_tm".

A first window appears. The image shows the current hologram on the camera, which contains the output of the fiber (see Figure 14). You can also observe this on the viewing card in filter holder FH3. Note that the fiber output is a random speckle pattern, even though at this point the fiber receives a plane wave illumination from the SLM on the other side.

A limited "region of interest" around the fiber should be selected at this point (red circle in Figure 14). This is to reduce the amount of data that needs to be processed during the measurement. If necessary, adjust the region of interest by using the arrow keys and the "+" and "-" keys on your keyboard. The region of interest should include the fiber speckle entirely, without too much extra margin. Press "ENTER" when done.

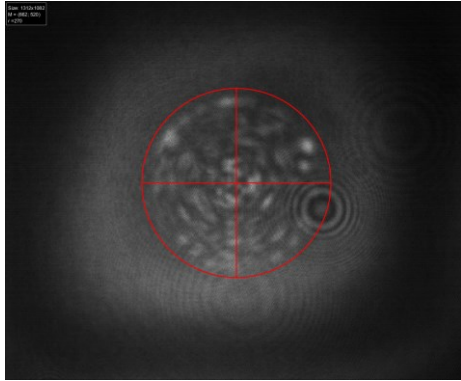


Figure 14: Selection of the region of interest around the fiber.

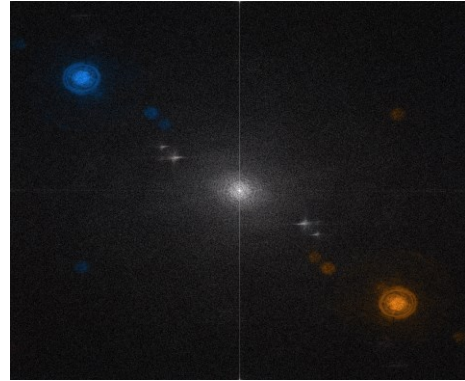


Figure 15: Selection of the correct order for the holographic reconstruction

A second window appears. It is the Fourier domain of the hologram currently recorded by the camera. Here, you have to select the region of the off-axis digital holograms that will be used for the holographic reconstruction. For your convenience, the square terms have been colored in white, the real order in orange, and the conjugate order in blue, as in Figure 15. Which order (real or conjugate) should be used to measure the transmission matrix? Why not the other one? Select the correct order with the "+", "-", and arrow keys. For the current fiber (50 μm core and NA 0.22), the radius of the region you select should be around 28 "Fourier-pixels". Press "ENTER" when done.

The measurement will start automatically. The SLM will quickly project a few thousand test patterns at the fiber input, and the camera will synchronously acquire and reconstruct the corresponding holograms at the output. During this procedure, do not lean on the optical table or interact with the setup in any way – interferometric measurements are very sensitive, and you may perturb the results.

- c) Open the script "script_tm_patterns_demo" and run it to draw a controlled output pattern through the fiber. You can view the patterns directly on the card in FH2. Block the reference beam by placing the cardboard back in FH2 in order to better see the pattern.

Record your results using the camera tool.

Optionally, you can try a few more output patterns. You may need to copy the code to a new script in order to change the pattern.

- d) While one of your patterns is being projected through the fiber, change the geometric shape of the fiber by using the positional adjuster PA. What happens with the output pattern? Explain briefly why this happens. How far approximately do you have to move the adjuster before you see a significant effect? Does the pattern return to its original state after you move back?

6. References

Reference [1] is the PhD thesis on which this lab session is based. Chapter 2 and 3 contain detailed information about holography, spatial light modulators and the transmission matrix.

Reference [2] is a classic textbook with detailed explanations about holography (chapter 9) and spatial light modulators (section 7.2).

Reference [3] has explanations about optical fibers in chapter 9.

- [1] D. Loterie, "Microscopy and digital light shaping through optical fibers," EPFL Thesis No. 7799, 2017, available at <http://infoscience.epfl.ch/record/229152>
- [2] J. Goodman, "Introduction to Fourier Optics", Roberts and Company Publishers, 2005, available at the EPFL library.
- [3] B. E. A. Saleh, M. C. Teich, "Fundamentals of Photonics", John Wiley & Sons, Inc., 2007.